Investigating Galaxy Data

Name: ____

Objective

The purpose of this assignment is to apply your knowledge of circular motion and gravity to analyze galaxy data and determine if there are any discrepancies between our observations and expectations.

1 What is a Model?

A model is a simplified representation of a system or phenomenon that helps us understand and predict its behavior. In science, models are essential tools for testing hypotheses and validating theories against observations.

Models are constructed based on known laws and principles and include key variables and relationships that define the system being studied. For example, in astronomy, we use models to describe the motion of planets, stars, and galaxies by applying Newton's laws of motion and gravitation.

A good model should:

- Capture the essential features of the system.
- Make accurate predictions that can be tested against experimental or observational data.
- Be as simple as possible while still being accurate.

However, no model is perfect. Models often make assumptions and simplifications that may not hold true in all situations. For instance, when modeling the orbits of planets, we might assume they are perfect circles, even though they are slightly elliptical in reality. These simplifications are necessary to make the problem manageable but can lead to discrepancies between the model's predictions and actual observations.

In this assignment, we will create a model for the rotational velocity of stars in a galaxy and compare it to observational data. By examining any discrepancies, we can explore potential reasons why our model might not perfectly match reality, leading to new insights and improvements in our understanding.

2 Gravity Review

Newton's Second Law states that the force on a mass, m, is related to its acceleration, a, through the equation:

$$F = ma \tag{1}$$

Additionally, the gravitational force between two masses, m_1 and m_2, is given by:

$$F_G = \frac{Gm_1m_2}{r^2} \tag{2}$$

where r is the distance between the two objects, and $G = 6.6743 \times 10^{-11}$ m^3·kg^{-1}·s^{-2} is the gravitational constant.

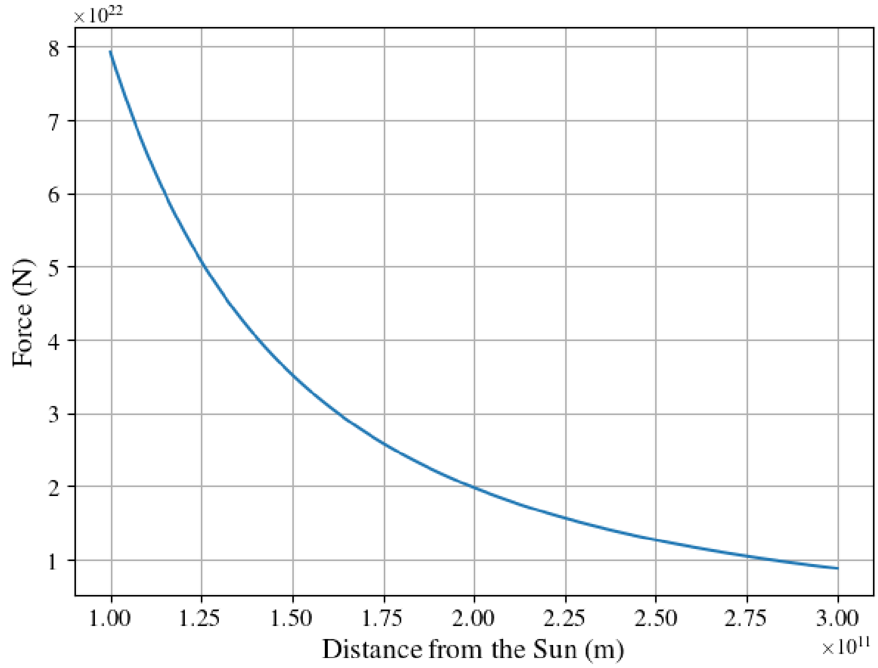
For example, if we use $m_1 = M_\odot = 1.989 \times 10^{30}$ kg (the mass of the Sun) and place a planet with the same mass as the Earth ($m_2 = M_E = 5.972 \times 10^{24}$ kg) at various distances from the Sun, we can see how the gravitational force between them decreases as the distance increases. In other words, we can use Equation 2 to model the force between the Sun and a planet with the same mass as Earth for different distances. This model is shown in Figure 1.

Sample calculation for $r = 1.25 \times 10^{11}$ m:

$$F_G = \frac{GM_\odot M_E}{r^2} = \frac{(6.6743 \times 10^{-11})(1.989 \times 10^{30})(5.972 \times 10^{24})}{(1.25 \times 10^{11})^2} \text{ N}$$

$$F_G = 5.074 \times 10^{22} \text{ N}$$

Figure 1: Modeling the force between the Sun and Earth.



3 Circular Motion

When an object undergoes circular motion with a radius, r, and a constant speed, v, it is actually accelerating towards the center of the circle. This acceleration is given by:

$$a = \frac{v^2}{r} \tag{3}$$

We can relate this equation to Newton's Second Law (Equation 1) to find the relationship between the centripetal force and the orbital velocity. According to Newton's Second Law:

$$F = ma \quad \text{and} \quad a = \frac{v^2}{r}$$

Therefore, the centripetal force, F_c, is:

$$F_c = \frac{mv^2}{r} \tag{4}$$

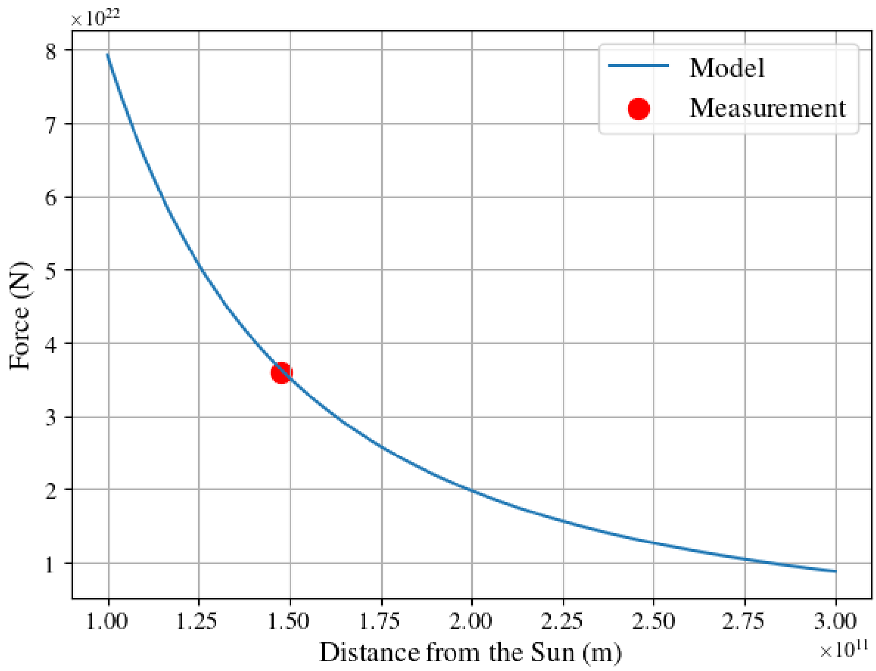
Let's apply this to the Earth-Sun system. The Earth is located approximately $r_E = 1.475 \times 10^{11}$ m from the Sun. Based on the time it takes to complete one orbit (365.25 days), we can determine the Earth's orbital speed, which is $v_{\text{meas}} = 29,780$ m/s. Using this orbital speed, we can calculate the centripetal force on Earth and compare it to the theoretical gravitational force using our model with $r = 1.475 \times 10^{11}$ m.

Calculating the "measured" force between the Sun and Earth:

$$F_{\text{meas}} = \frac{M_E \cdot v_{\text{meas}}^2}{r_E} = \frac{(5.972 \times 10^{24})(29,780)^2}{(1.475 \times 10^{11})} \text{ N}$$

$$F_{\text{meas}} = 3.591 \times 10^{22} \text{ N}$$

Figure 2: The force between the Sun and Earth.



The calculated force is quite close to the theoretical value!

To quantify this closeness, we use the percentage error formula:

$$\%_{error} = \frac{|F_{\text{meas}} - F_{\text{model}}|}{F_{\text{model}}} \times 100\% \tag{5}$$

Calculating the percentage error:

$$\%_{error} = \frac{|3.591 \times 10^{22} - 3.644 \times 10^{22}|}{3.644 \times 10^{22}} \times 100\%$$

$$\%_{error} = 1.45\%$$

While this error is quite small, we can hypothesize why our model does not perfectly match the measurement. For instance:

- The Earth's orbit is not perfectly circular, so $F_c = \frac{mv^2}{r}$ is not an exact model.
- Large planets, like Jupiter, also have a substantial gravitational interaction with Earth, so $F_G = \frac{Gm_1m_2}{r^2}$ is not exact either.

This example should give you some guidance on how to:

1. Create a model of a physical phenomenon.
2. Compare your measurements to that model.
3. Use this comparison to hypothesize what might be missing from your model.

4 Galaxies

Similar to how the Earth orbits around the Sun, stars in a galaxy rotate around the center of the galaxy. A rotation curve is a plot of the rotational (or orbital) velocity of stars in a galaxy as a function of their distance from the center (radius).

Using photometric data of the luminous matter, we can create a rotation curve of a galaxy. This curve is used to estimate the enclosed mass within a certain radius by equating the centripetal force to the gravitational force. We assume that, like the Earth orbiting the Sun, stars have circular orbits. Therefore, we can use the same relationship as before:

$$\frac{mv^2}{r} = \frac{GmM_{enc}(r)}{r^2} \tag{6}$$

where:

- v = rotational velocity
- G = gravitational constant
- m = mass of the star
- $M_{enc}(r)$ = enclosed mass as a function of radius (note: this is not the same as $M_{enc} \times r$)
- r = radius or distance from the center of the galaxy

Since stars are very massive objects and the distances we consider are also very large, we typically use units of solar masses ($1M_\odot = 1.989 \times 10^{30}$ kg) and kiloparsecs (1kpc $= 3.086 \times 10^{19}$ m). We can also express the gravitational constant in these units, where $G = 4.30 \times 10^{-6}$kpc · km$^2/M_\odot/$s^2.

Observations of galaxies show that the luminous matter is largely contained within the central "bulge." Therefore, at larger distances, if we assume that $M_{enc} = M_{bulge}$, Equation 6 simplifies to:

$$\frac{mv^2}{r} = \frac{GmM_{bulge}}{r^2} \tag{7}$$

5 Activity Part 1: Creating a Rotational Model

Your goal for this activity is to:

1. Use the relationship in Equation 7 to create a model for the rotational velocity of a star in a galaxy in terms of the mass enclosed within that star's orbit.
2. Compare this model to measured data from the provided data tables.
3. Discuss whether or not your model adequately represents the data.

5.1 Creating a model

a) Using Equation 7 and your algebra skills, develop an equation for v that will serve as **your model** for the rotation curve of your galaxy.

To do this, follow these steps:

1. Start with Equation 7:

$$\frac{mv^2}{r} = \frac{GmM_{bulge}}{r^2}$$

2. Simplify the equation to solve for v. This will give you a model for the rotational velocity as a function of radius r and mass M_{bulge}.

5.2 Galaxy data

NGC7814, also known as the "Little Sombrero," is a spiral galaxy located approximately 40 million light-years away in the constellation Pegasus. It is characterized by a prominent central bulge and a thin, flat disk. The galaxy's edge-on orientation provides a clear view of its structure, making it an excellent subject for studying galactic dynamics and mass distribution.

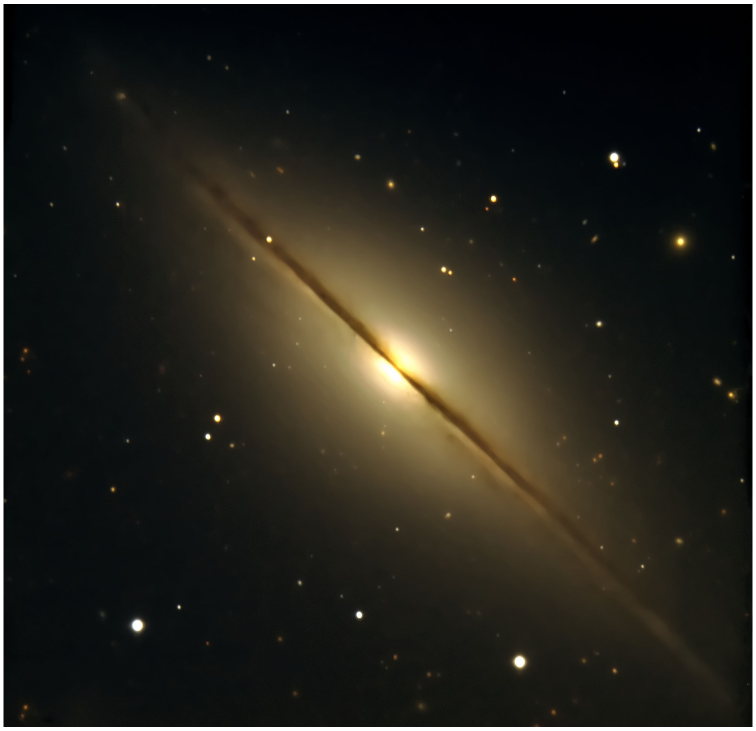


Figure 3: NGC7814 - The Little Sombrero Galaxy

The table below provides some **measurements** obtained from NGC7814 of the rotational velocities (v) of stars at different distances from the center of the galaxy (r).

Galaxy Name	NGC7814
$\mathbf{M_{bulge}}$	$0.5912 \times 10^{10}\ M_\odot$
r (kpc)	v_{meas} **(km/s)**
0.6300	265.0000
2.7900	233.0000
5.0300	228.0000
7.2600	227.0000
9.5000	223.0000
11.7400	216.0000
14.0100	214.0000
16.1800	214.0000
18.4400	214.0000

5.3 Plotting the data

For this activity, you can either use a graphing program of your choice or do the calculations and graphs by hand.

a) Use the data table for NGC7814 to create a rotation curve plot (i.e., plot the measurements for v against r) with appropriate axis labels and units.

b) On this same plot, include a curve that represents your model. To accomplish this, follow these steps:

1. Use your model equation to calculate the theoretical rotational velocity for various values of r.

2. Fill in the table below with your calculated velocities.

3. Plot these points on your graph and connect them with a smooth curve.

Show at least one sample calculation.

r (kpc)	v (km/s)

5.3.1 Sample Calculations

5.3.2 Results

a) Calculate the percentage error between the measurements and your model for each star, then calculate the average percentage error. Show at least one sample calculation.

Sample percentage error calculation:

$$\%_{error} = \frac{|v_{meas} - v_{model}|}{v_{model}} \times 100\%$$

b) Write down what you observe about the agreement between the measurements and your model. What can you say about the measured rotational velocities at large distances from the galaxy's center?

c) Does your model appear to accurately represent the measurements? Use evidence from your comparisons to justify your answer.

d) What do you think might be missing from your model? (Hint: what assumptions did we make?)

6 Enclosed Mass

Previously, we modeled the rotational velocity of a star orbiting its galaxy using the equation:

$$v = \sqrt{\frac{GM_{enc}}{r}} \tag{8}$$

where:

- v = rotational velocity
- $G = 4.30 \times 10^{-6}$ kpc · km$^2/M_\odot/$s^2 (the gravitational constant)
- M_{enc} = enclosed mass as a function of radius
- r = radius or distance from the center of the galaxy

However, when we assumed that the enclosed mass was simply the mass of the bulge of the galaxy, the model did not accurately represent the experimental data. This discrepancy leads us to explore how we can more accurately estimate the enclosed mass.

In reality, multiple sources contribute to the mass of a galaxy. Some of the main sources include:

1. The central black hole

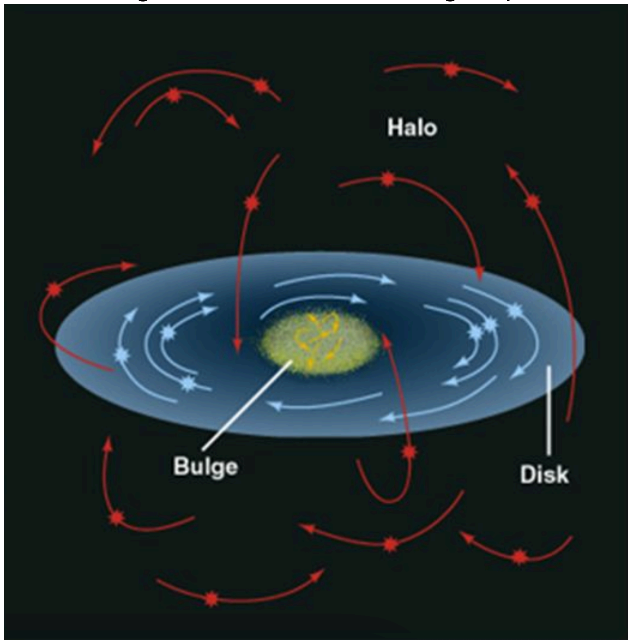
2. The bulge of the galaxy

3. The main stellar disk

4. The surrounding gas cloud

Our goal is to model the enclosed mass by including these contributions and compare this model to the measured enclosed mass obtained from the radial velocity curve of the galaxy.

Figure 4: The structure of a galaxy.



To begin, our model will include these four contributions as follows:

$$M_{enc}(r) = M_{BH} + M_{bulge} + M_{disk} + M_{gas}(r) \tag{9}$$

where:

- M_{BH} = mass of the central black hole
- M_{bulge} = mass of the galaxy bulge
- M_{disk} = mass of the main stellar disk
- $M_{gas}(r)$ = mass of the gas cloud **as a function of radius**

The measurements we will be analyzing are from stars outside of the bulge and main stellar disk. Therefore, M_{bulge} and M_{disk} can be determined from their average spherical densities, ρ_{bulge} and ρ_{disk}, using the formula:

$$\text{Mass} = \text{Volume} \times \text{Density} = \frac{4}{3}\pi r^3 \rho \tag{10}$$

Equation 10 can also be used for M_{gas}, but note that this mass will continue to increase as we move further from the center of the galaxy.

7 Activity Part 2: Creating a Model

Your goal for this activity is to:

1. Use the relationship in Equation 9 to create a model for the enclosed mass within a given radius of the galaxy.
2. Compare this model to measured data from the provided data tables.
3. Discuss any discrepancies between the model and the measured data.

Table 1: Components of the enclosed mass for NGC 7814.

Galaxy Name	NGC7814
M_{BH}	$1.00 \times 10^8\ M_\odot$
ρ_{bulge}	$1.524 \times 10^8\ M_\odot/\text{kpc}^3$
r_{bulge}	2.10 kpc
ρ_{disk}	$5.592 \times 10^7\ M_\odot/\text{kpc}^3$
r_{disk}	6.05 kpc
ρ_{gas}	$7.329 \times 10^4\ M_\odot/\text{kpc}^3$

Table 2: Rotational curve data for NGC 7814.

r (kpc)	**v (km/s)**
6.14	228
7.26	227
8.37	226
9.50	223
10.65	219
11.74	216
12.82	214
14.01	214
15.09	214
16.18	214
17.36	214
18.44	214
19.53	214

7.1 Creating and Plotting the Model

a) Expand Equation 9 to create a **model** for M_{enc} in terms of r, M_{BH}, ρ_{bulge}, r_{bulge}, ρ_{disk}, r_{disk}, and ρ_{gas}. You can follow the steps below:

1. Start with Equation 9:

$$M_{enc}(r) = M_{BH} + M_{bulge} + M_{disk} + M_{gas}(r)$$

2. Use the provided values in Table 1 to express M_{bulge}, M_{disk}, and $M_{gas}(r)$ in terms of their densities and radii. Remember that the mass of a spherical region is given by:

$$\text{Mass} = \text{Volume} \times \text{Density} = \frac{4}{3}\pi r^3 \rho$$

3. Substitute these expressions into Equation 9 to get a full model for $M_{enc}(r)$.

b) Use your model and the data in Tables 1 and 2 to determine the enclosed mass within each star's orbit.

Show at least one sample calculation for the model value of the enclosed mass ($M_{enc}(r)$).

c) To obtain the measured enclosed mass from the rotational curve data, rearrange Equation 8 and use the measured velocities in Table 2.

Show at least one sample calculation for the measured enclosed mass.

Table 3: Enclosed Mass Data for NGC7814

r (kpc)	M_{enc} **(model,** $M_{\odot}$**)**	M_{enc} **(measured,** $M_{\odot}$**)**
6.14		
7.26		
8.37		
9.50		
10.65		
11.74		
12.82		
14.01		
15.09		
16.18		
17.36		
18.44		
19.53		

d) In the graph area on the following page (or with your graphing software), plot both the model and measured enclosed mass vs. the radius. Include appropriate axis labels and legends.

7.1.1 Sample Calculations

7.1.2 Results

a) At $r = 19.53$ kpc, how much enclosed mass is missing from your model compared to the measured value?

b) Write down what you observe about the trend of the model. What about the measurements?

c) Does your model appear to accurately represent the measurements? Use evidence from your comparisons to justify your answer.

d) What do you think might be missing from your model? Be creative!!

8 Self-Assessment

For each statement, circle the answer that best represents how you feel about today's lesson.

a) After today's lesson, I have an understanding of what a "model" is.

Strongly Agree | Agree | Disagree

b) After today's lesson, my confidence in my ability to compare measurements to a model ___________.

Improved | Did not improve